4. Introduction antennas

4.1 Power Density, Poynting vector

4.1.1 Power Density

The power radiated per unit area is $\overline{P}=\overline{E}\wedge\overline{H}$. Let's call $P_\Omega(\theta,\varphi)$ the power-density in a direction as the power per unit space-angle, then is the totally radiated power

$$W_r = \int_\Omega P_\Omega(\theta,\phi)\, d\Omega \quad with \quad d\Omega = \sin\theta d\theta d\phi \qquad (4.1)$$

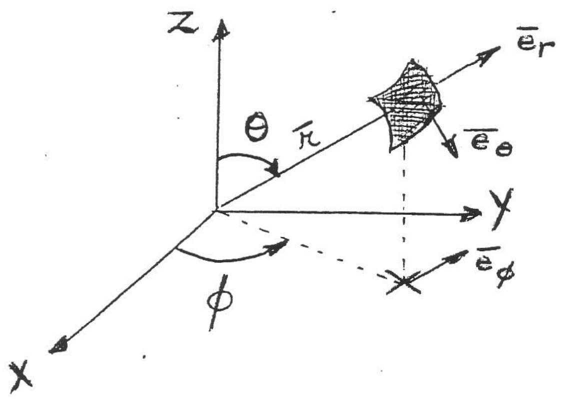


Fig.4-1: Coordinates.

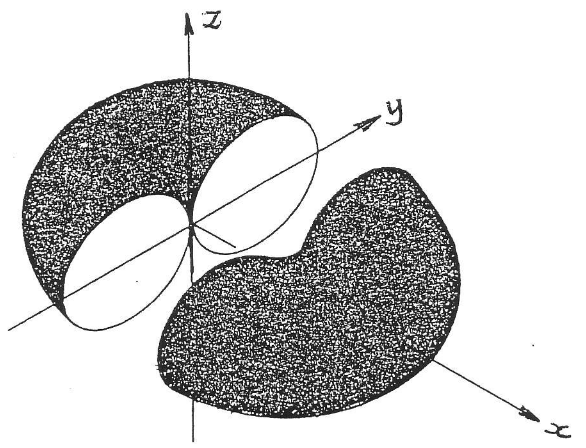
The average radiated power density per unit space-angle becomes

$$P_{\Omega av} = \frac{W_r}{4\pi} \qquad (4.2)$$

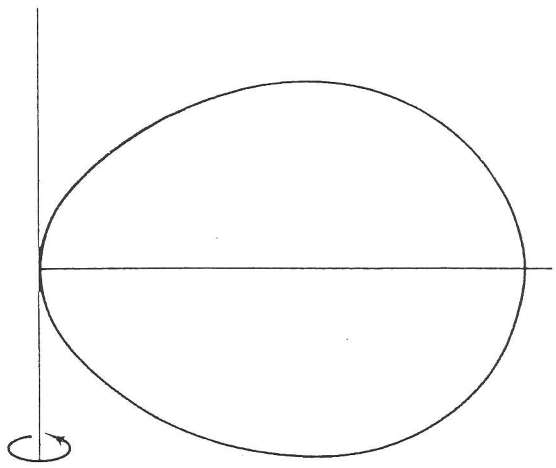
4.1.2 Poynting vector

The antenna-radiation-diagram is the antenna-power-diagram which indicates the power density flowing through a sphere with radius r. This variation can be shown in the form of a polar plot in which the radial vector has the magnitude of P_a, power density antenna, for a fixed distance from the source and the direction is that of the point at which P_a is measured.
To avoid specifying any particular distance it is usual to display P_a/P_m, P_m being the maximum radiated power density in any direction. Such a polar plot is reffered to as **radiation pattern** of the antenna. The locus of P_a/P_m describes a surface, an example is shown in fig.4-2(a) for the short electric dipole. Several variations are useful.

For example, the radiation pattern of the short dipole is symmetrical about the axis, so it is only necessary to show the single plot of fig.4-2(b). The radiation of such an antenna may also be displayed in Cartesian form as in fig.4-2(c).



(a)



(b)

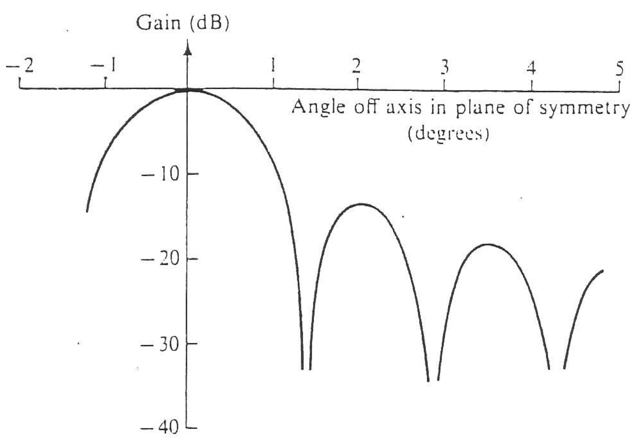


Fig.4-2: Radiation pattern of a short dipole.(a) Three-dimensional plot.(b) Section.(c) relative radiation pattern for a high-gain antenna.

This diagram corresponds with the average **Poynting-vector**

$$\langle \overline{P} \rangle = \langle \overline{E} \wedge \overline{H} \rangle \quad (4.3)$$

or in sinusoidal terms

$$P_{av} = \frac{1}{2} R_e (\underset{\bullet}{\overline{E}} \wedge \underset{\bullet}{\overline{H}^*}) \quad (4.4)$$

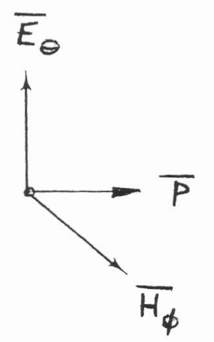


Fig.4-3: Poynting-vector.

4.2 Impedance, Directivity, gain, effective surface and height antenna factor

4.2.1 Impedance

The equivalent circuit of an antenna and transmitter/receiver is given below

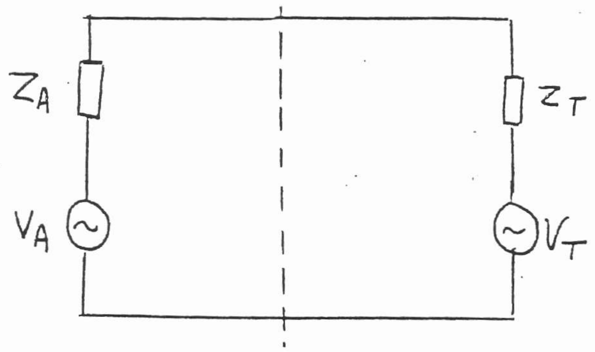


Fig.4-4: Equivalent circuit.

With $Z_A=R_L+R_r+jX_A$ and R_L= loss resistance

$Z_t=Z_A+Z_T$ R_r= radiation resistance

$Z_T=R_T+jX_T$ X_A= antenna reactance

The (matching) condition for maximum energy-transfer is

$$Z_A=Z_T^*$$

When the above system works as *receiver*-antenna we have $V_T=0$

- power delivered at the receiver

$$W_R = I^2.R_T = (\frac{V_A}{|Z_t|})^2.R_T \quad (4.5)$$

- power re-radiated by receiver-antenna

$$W_s = I^2.R_r = (\frac{V_A}{|Z_t|})^2.R_r \quad (4.6)$$

- power dissipated in ohmic (Joule) losses of the antenna

$$W_L = I^2.R_L = (\frac{V_A}{|Z_t|})^2.R_L \quad (4.7)$$

When the system works as *transmitter*-antenna, we have V_A=0

- power radiated by antenna

$$W_r = I^2.R_r = (\frac{V_T}{|Z_t|})^2.R_r \quad (4.8)$$

- power dissipated in ohmic (Joule) losses of the antenna

$$W_L = I^2.R_L = (\frac{V_T}{|Z_t|})^2.R_L \quad (4.9)$$

In practice the antenna is matched only in a small frequency-range, a range for which VSWR≤2.

4.2.2 Directivity and Gain

We define the directivity-function as

$$d(\theta,\phi) = \frac{P_\Omega(\theta,\phi)}{P_{\Omega av}} = \frac{4\pi P_\Omega(\theta,\phi)}{W_r} \quad (4.10)$$

The maximum directivity is

$$D = \frac{P_{\Omega max}}{P_{\Omega av}} = \frac{4\pi P_{\Omega max}}{W_r} \quad (4.11)$$

We didn't take into consideration the losses of the antenna, so now we define the gain-function and the effective gain. The gain-function is the variation of the radiated power (transmitter-power W_T=W_r+W_L) with respect to the angles θ and ψ.

$$g(\theta,\phi) = 4\pi \frac{P_\Omega(\theta,\phi)}{W_T} \qquad (4.12)$$

and the gain becomes

$$G = \frac{4\pi P_{\Omega max}}{W_T} \qquad (4.13)$$

The efficiency of an antenna becomes

$$\eta = \frac{W_r}{W_T} = \frac{G}{D} \qquad (4.14)$$

with 50% < η < 75%.

We can determine the gain of an antenna in any direction by measuring the power density in the far field in that direction for a given total transmitted power density. The power density can be determined using a standard receiving antenna with known gain, such as the simple half-wave dipole.

4.2.3 Effective surface

$$A_{eff} = \frac{\textit{received max power (matched)}}{\textit{power-density incident wave}} = \frac{W_{r,\max}}{P} \qquad (4.15)$$

If a matched antenna is used, we have $R_L+R_r+jX_A=R_T-jX_T$, so considering

$$W_r = \left(\frac{V_A}{|Zt|}\right)^2 . R_T$$

$$W_r = \frac{V_A^2 . R_T}{4(R_L+R_r)^2} \approx \frac{V_A^2}{4R_r} \quad (R_T \approx R_r, \textit{no losses})$$

$$\textit{this means that } A_{eff} = \frac{V_A^2}{4.R_r.P} \qquad (4.16)$$

One can proove for all antennas:

$$A_{eff} = \frac{\lambda^2 . G}{4\pi} \qquad (4.17)$$

is the effective receiving area of an antenna.

Now we can examine a one-direction radio transmission formule.

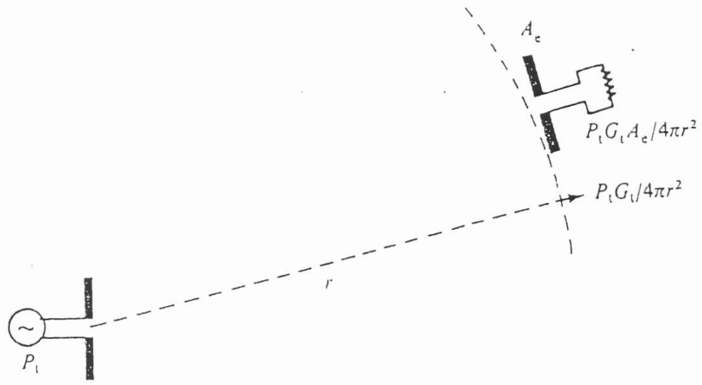


Fig.4-5: A transmitter-receiver system.

Consider a transmitter with gain G_T, transmitting a total power W_T watts, as shown in fig.4-5. Then the power density P, at distance r from the antenna, is

$$P_r = \frac{W_T . G_T}{4\pi r^2} \quad [\frac{W}{m^2}]$$

A receiving antenna of effective area A_{eff} and power gain G_R at this point, directed towards the transmitter, produces an available power $P_R . A_{eff}$ given by

$$W_R = \frac{G_T . W_T . A_{eff}}{4\pi r^2}$$

Taking in consideration the losses of the medium L_m, and with some losses caused by polarisation-rotation L_p, we receive the following power

$$W = \frac{W_T . \lambda^2 . G_R . G_T . L_m . L_p}{(4\pi r)^2}$$

with G_T and G_R the transmitter and receiver gains.
Suppose that the transmitter is located on place 1 and the receiver on place 2, we get

$$W_R = \frac{W_T . Aeff2 . L_m . L_p . G_1}{4\pi r^2}$$

$$G_1 . A_2 = \frac{W_R . 4\pi r^2}{W_T . L_m . L_p}$$

Changing the position of receiver and transmitter, we get

$$G_2 . A_1 = \frac{W_R . 4\pi r^2}{W_T . L_m . L_p}$$

So when medium and antennae are reciprocal we have for all antenna-types

$$\frac{G_1}{A_1} = \frac{G_2}{A_2}$$

It follows that the ratio of power gain to receiving area is the same for any antenna.

4.2.4 Effective antenna height

The induced voltage V_A as a result of incident waves with field strength E is V_A=E.h. The corresponding power density is

$$P = \frac{E^2}{Z_o} \quad with \ Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = = 120.\pi \qquad (4.18)$$

Z_o is the impedance of free space. Further we have

$$A_{eff} = \frac{W_{R,\max}}{P} = \frac{V_A^2}{4R_r P} = \frac{E^2 h^2}{4R_r P} = \frac{E^2 h^2 Z_o}{4R_r E^2} = \frac{h^2 Z_o}{4R_r} \qquad (4.19)$$

The corresponding 'h' is called the effective height antenna factor h_{eff}.

4.3 Effective isotropic radiated power

The radiation pattern defined in the previous chapter describes the directional properties of the radiated field. The absolute magnitude is described by *comparing* the actual field with the field produced at a similar point by a standard reference antenna transmitting the same total power. The reference antenna is usually taken as the fictional **isotropic radiator**, for which the input power is radiated uniformly in all directions. For the isotropic antenna we have

$$P_\Omega(\theta,\phi) \doteq r^2 |\overline{E} \wedge \overline{H}| = C^{st} \qquad (4.20)$$

and power density

$$P_r = \frac{W_T}{4\pi r^2} = \frac{E^2}{Z_o} \qquad (Z_o=120\pi)$$

4.4 Polarisation

This is the geometrical orientation of the top of the time-dependent electrical field-vector E. In practice the polarisation of the radiated energy changes with respect to place, so many parts of the radition-pattern can have a different polarisation. We can discern linear, circular and elliptical polarisation.

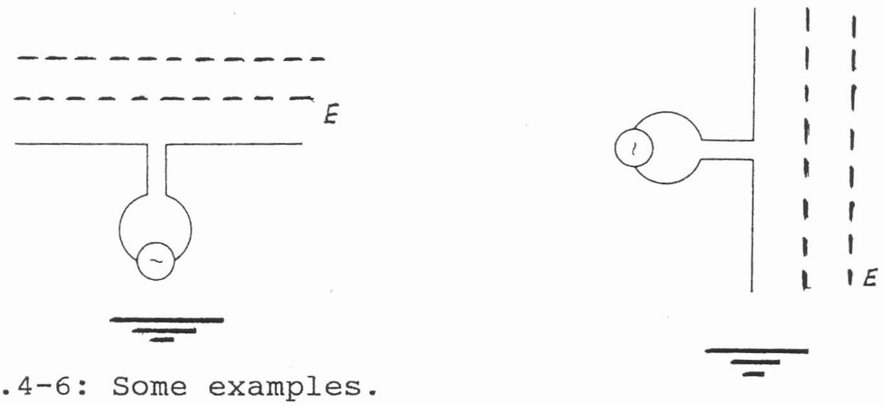


Fig.4-6: Some examples.

Linear E-(far) field
Horizontal and vertical

II. ANTENNAS

1. Impedance matching networks

If an aerial feeder is used to deliver power to the aerial with minimum loss, it is necessary for the load to behave as a pure resistance equal in value with the characteristic impedance of the line. Under these conditions no energy is reflected from the point where the feeder is joined to the aerial, and in consequence no standing waves appear on the line. When the correct terminating resistance is connected to any feeder, the voltage and current distribution along the line will be uniform.

1.1 Baluns (balanced → unbalanced)

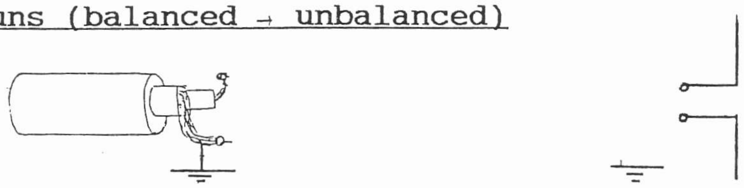


Fig.1-1: Unbalanced (assym.) output -- balanced (sym.) input.

In most cases an aerial requires a balanced feed with respect to ground, and therefore it is necessary to use a device which converts the unbalanced output of a coaxial cable to a balanced output as required by the aerial. This device also prevents the wave which has been contained within the cable from tending to 'spill over' the extreme end and travel back over the surface of the cable. Whenever this occurs there are two important undesired effects; firstly, the re-radiated wave modifies the polar diagram of the attached aerial, and secondly the outer surface of the cable is found to have a radio frequency voltage on it.

To prevent this, a <u>balance to unbalance transformer</u> is connected between the feeder cable and the aerial. The simplest balun consists of a short circuited quarter-wave section of transmission line attached to the outer braiding of the cable as shown in fig.1-2.

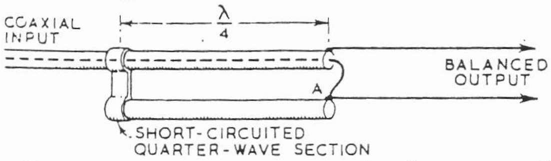


Fig.1-2: Quarter-wave open balun or Pawsey stub.

At the point A the quarter-wave section presents a very high impedance which prevents the wave from travelling over the surface. The performance of this device is, of course, dependent upon frequency, and its bandwidth may have to be considered in the design. Several modifications of the simple balun are possible.

For example, the single quarter-wave element may be replaced by a quarter-wave coaxial sleeve, thus reducing radiation loss, see fig.1-3.

To prevent the ingress of water and to improve the mechanical arrangement, the centre conductor may itself be connected to a short-circuited quarter-wave line acting as a 'metallic insulator' as shown in fig.1-4.

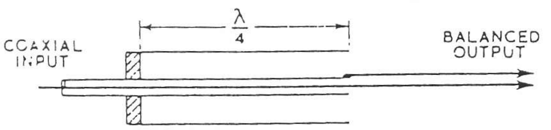


Fig.1-3: Coaxial sleeve balun.

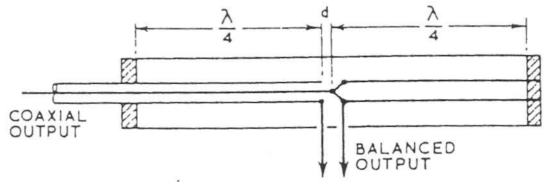


Fig.1-4: Totally enclosed coaxial balun. The right-hand section acts as a metal insulator.

A useful variation is that shown in fig.1-5 which gives a 4:1 step-up of impedance. The half-wave loop is usually made from flexible coaxial cable, and allowance must therefore be made for the velocity factor of the cable when calculating a half-wavelength. It may be inconvient at frequencies above about 2000MHz to mount the coaxial sleeve balun close to a dipole radiator.

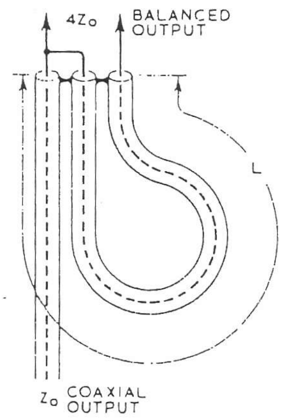


Fig.1-5: A coaxial balun giving a 4:1 impedance step-up. The length L should be $\lambda/2$ according with the velocity factor of the cable. The outer braiding may be joined at the indicated points.

We can also use a HF-transformer to convert asymmetrical coax to symmetrical feeders.

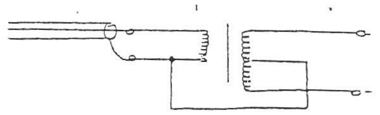


Fig.1-6: High-frequency transformer as balun.

The use of broadband ferrites permittes us to make a broad-band-balun.

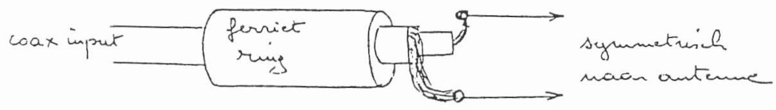


Fig.1-7: balun with ferrite-ring.

The ring is pushed on the end of the coax-cable. Return-currents which want to flow on the outside (shield) of the coax are prevented by the ferrite to flow back because they see a high-impedance. The ferrite (high permeabillity desirable) acts as an inductance (impedance). When the permeabillity is high for a large range of frequencies, we can construct broad-band-baluns.

1.2 Narrowband matching networks

The term matching is used to describe the procedure of suitably modifying the effective load impedance to make it behave as a resistance and to ensure that this resistance has a value equal to the characteristic impedance of the feeder used. To make a complete load (ie a load posessing both resistance and reactance) behave as a resistance, it is necessary to introduce across the load a reactance of equal value and opposite sign to that of the load, so that the reactance is effectively 'tuned out'. A very convenient device which can theoretically give reactance values from the minus infinity to plus infinity, (ie pure capacitance to pure inductance) is a section of transmission line either of length variable between zero and one half-wavelength with an open-circuited end or alternatively of length somewhat larger than one half-wavelength with an adjustable short-circuit capable of being adjusted over a full half-wavelength.

1.2.1 Stub tuners

· example: matching with 1 stub

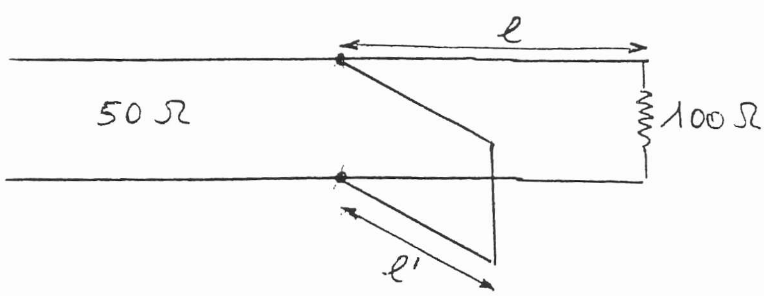


Fig.1-8: Possible circuit to match.

With a parallel stub we can only change the reactance part of an admittance.

With a matched circuit the (normalised) impedance and also the (normalised) admittance must be 1!
So on the position where the stub will be placed, the real part of the admittance must be 1! As said a parallel stub only changes the reactance part. We can calculate the different parameters or use a Smith-chart.

<u>calculation</u>: suppose Y'_L = real
at the position l, measured from load in the direction of the source, where the stub will be placed, is valid

$$Y'(l) = \frac{Y_L + j\,tgkl}{1 + jY_L\,tgkl} \quad with \quad Re[Y'(l)] = (must\ be)\ 1!$$

$$\Rightarrow Y_L(1+tg^2kl) = 1+Y_L^2 tg^2kl \Rightarrow l = \pm\frac{\lambda}{2\pi}Bgtg\sqrt{\frac{1}{Y_L}}$$

Now the stub must compensate the reactance part of Y'_L.
One can prove that

$$we\ can\ prove\ Im[Y'(l)] = \pm(1-Y_L)\,.\frac{1}{\sqrt{Y_L}}$$

We can calculate this again for complex loads (as antenna-impedances are in reality) but the formules will get more complex. To avoid this one can use the Smith-chart!
For the example above we find with the calculations:
$Y'_L=0.5$; $l=0.1520\lambda$; $Im[Y'(l)]=+-0.707$; $l'=0.152\lambda$;

<u>Smith-chart</u>: Y'(x) is on the circle $|K|=1/3$. The stub must be placed where Re[Y'(x)]=1, so we move us along the K-circle until Re'=1 (direction of source). We find X'=0.707.
See fig.1-9.
Linking the origin (of the chart) with this point gives, seen from Y'_L, a rotation over an electrical length of 0.152λ. We compensate with a stub with $Y'_{stub}=-0.707$, corresponding with a length $l'=0.152\lambda$.
This is one possibility! We could also use X'=-0.707 on $l=0.35\lambda$, and stub $Y'_{stub}=+0.707$ with $l'=0.35\lambda$.

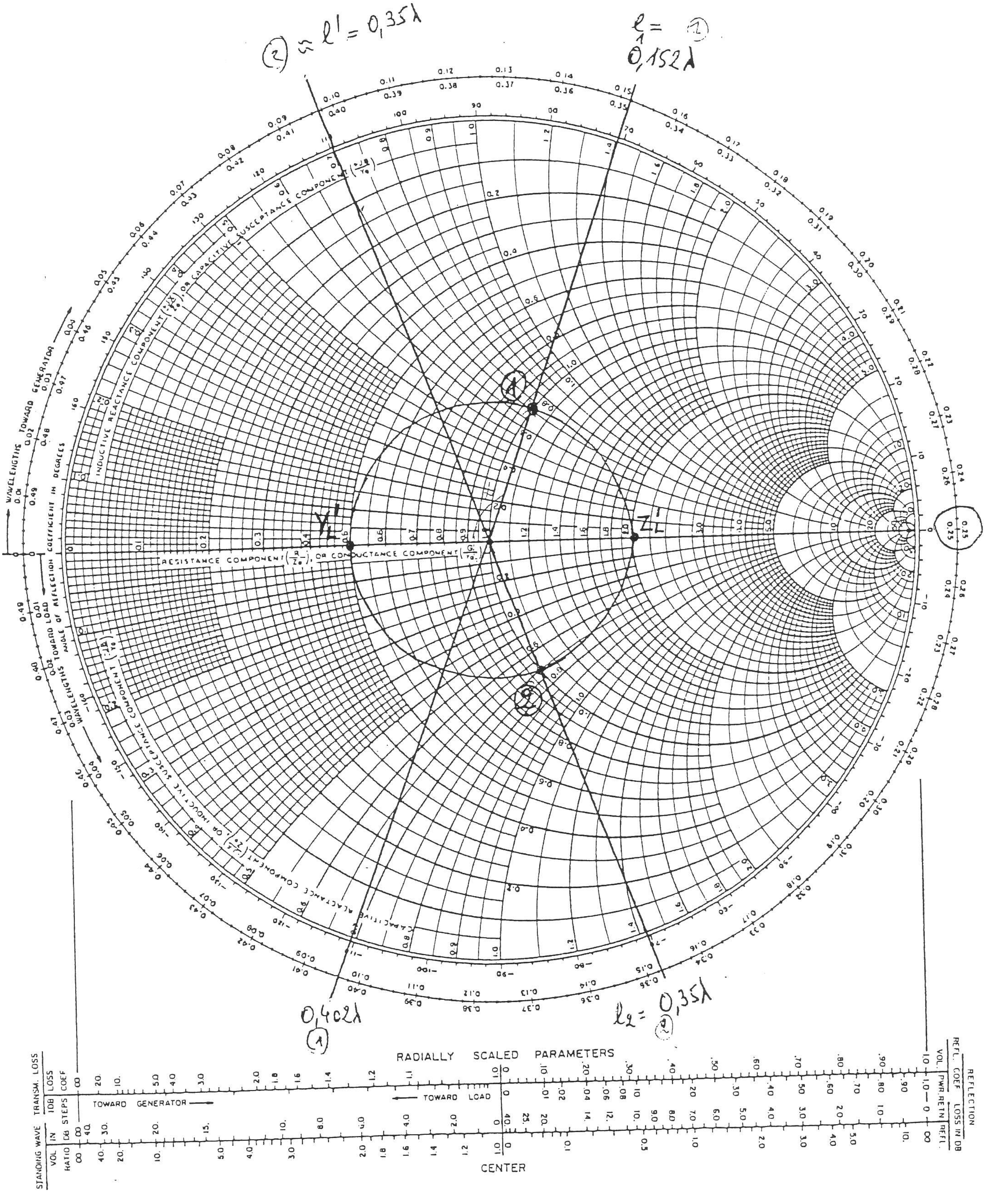


Fig.1-9: Smith-chart use for 1 stub.

• *example:* matching with 2 stubs

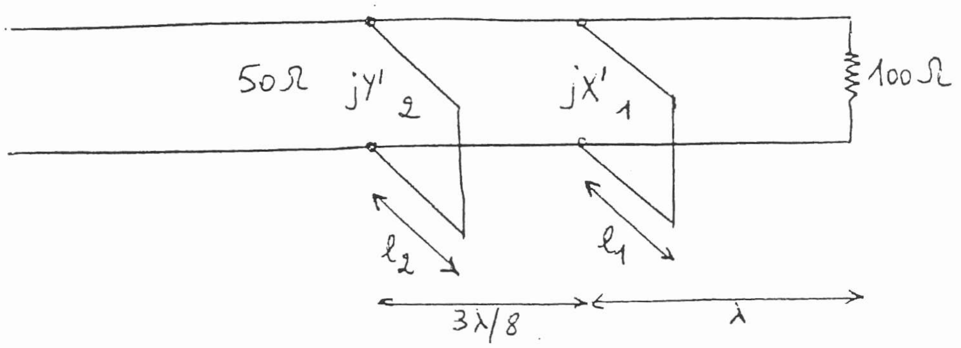


Fig.1-10: Matching with 2 stubs.

The two stubs are always placed on a distance $3\lambda/8$ from each other. One stub is always placed on the load, or on a dis tance of some half-wavelengths from it (for ex. λ). At point 2 $Y'_2=1$; so with stub 2 we can (only) change the reactive part. Stub 1 must realize a certain admittance giving real part =1 over a distance $3\lambda/8$ in the direction of the source. See fig.1-11. $3\lambda/8$ is a rotation over 270°, so rotate the circle Re=1 over 270° to the load and take the intersections with Re=0.5. In our example there are 2 possibilities

$$Y_1a = Y_L+Y_{stub1_a}=0.5-j0.14,\ so\ Y_{stub1_a}=-j0.14$$
$$Y_1b = Y_L+Y_{stub1_b}=0.5-j1.9,\ so\ Y_{stub1_b}=-j1.9$$

Transformation over $3\lambda/8$ to the generator gives

$$Y_{2a} = 1-j0.7,\ so\ Y_{stub_{2a}}=+j0.7$$
$$Y_{2b} = 1+j2.9,\ so\ Y_{stub_{2b}}=-j2.9$$

The 2 (possible) solutions are:

1) stub(1) with length $L=0.228\lambda$ ($Y'_{1a}=0.5-j0.14$), at (2) we get $Y'_{2a}=1-j0.7$; This admittance gets real by adding a stub of j0.7 at point (2) with length 0.347λ.

2) stub(1) with length $L=0.078\lambda$ ($Y'_{1b}=0.5-j1.9$), at (2) we get $Y'_{2b}=1+j2.9$; Becomes real at (2) by adding a stub -j2.9 with length 0.053λ.

IMPEDANCE OR ADMITTANCE COORDINATES

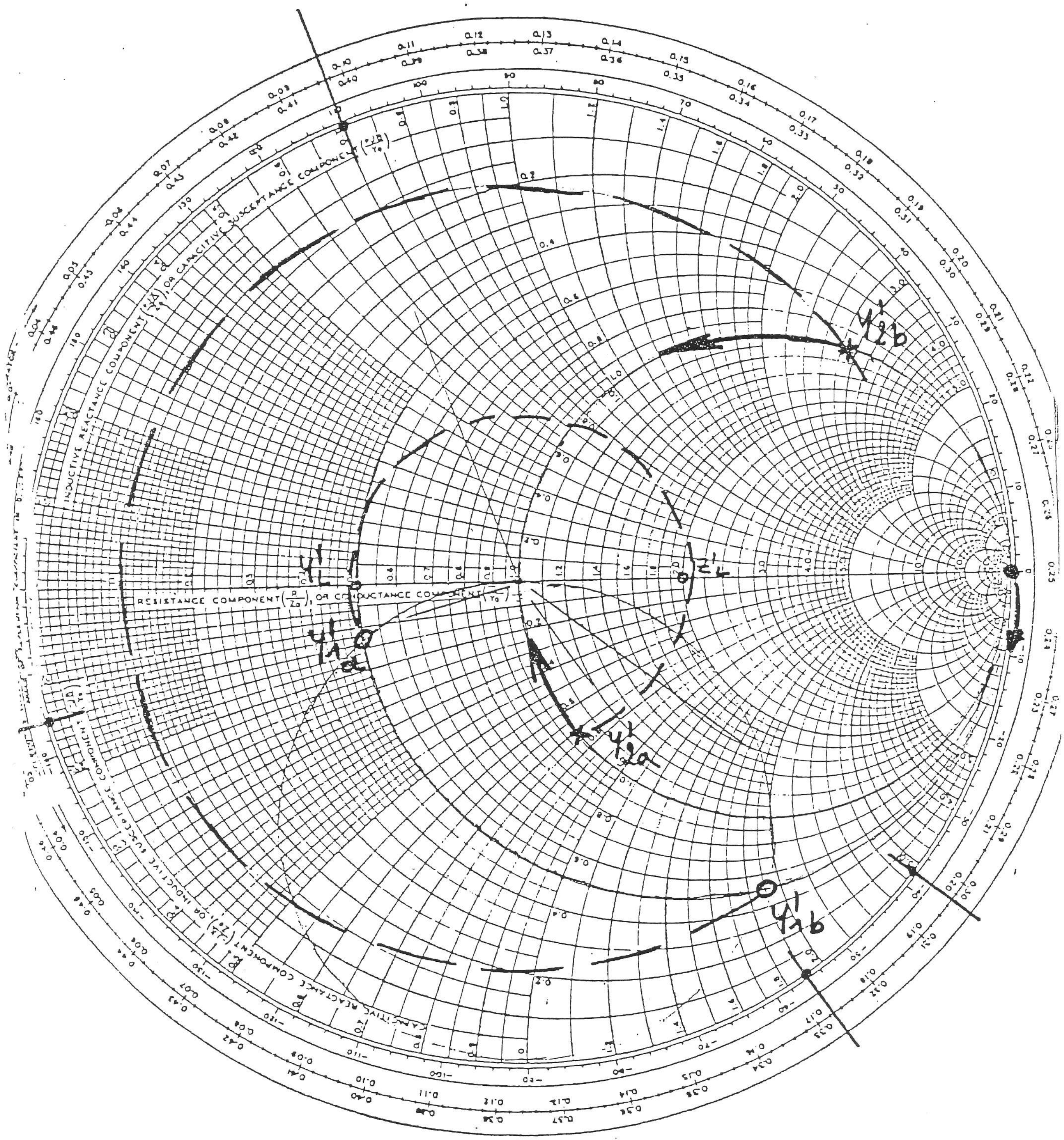


Fig.1-11: Smith-chart use for 2 stubs.

1.2.2 Matching with localized components

When one works in the HF-band (2-30 MHz) we can use discrete components for the ATU (automatic tuning unit), because stub-lengths would become to large.

- *example:* matching network with discrete components.

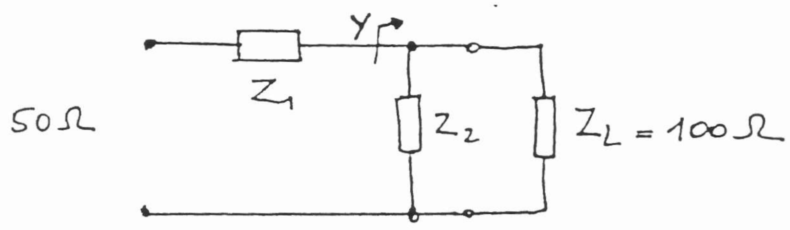


Fig.1-12: matching network with discreet components.

Normalise to 50Ω, so $Z'_1+(Z'_2\| Z'_L)=1$.
Z_1 and Z_2 are pure reactances, to match $\mathrm{Re}\{Z'_2\| Z'_L\}=1$.
the imaginary component of $(Z'_2\| Z'_L)$ is reduced to 0 with Z'_1.

Again there are 2 solutions (see fig.1-15):

1) $\mathbb{C}'_a=0.5/\omega$; $\mathcal{L}'_a=1/\omega$

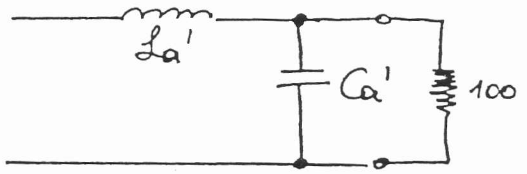


Fig.1-13: first possible network.

2) $\mathcal{L}'_b=2/\omega$; $\mathbb{C}'b=1/\omega$

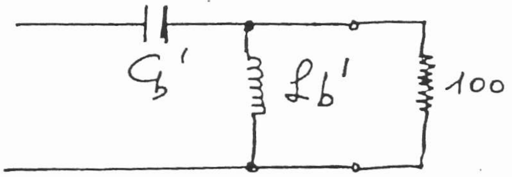


Fig.1-14: second possible network.

2. Isotropic antennas

2.1 Radiation pattern - power density

An isotropic antenna is a fictive (non realisable) antenna with an isotrope radiation-diagram, giving a constant power flux on a sphere with radius r.

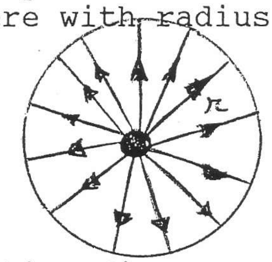


Fig.2-1: Radiation-diagram of an isotropic antenna.

For such an antenna is valid

$$P_\Omega(\theta,\phi) = \div r^2 |\overline{E} \wedge \overline{H}| = cst \quad (2.1)$$

The power density becomes

$$P_r = \frac{W_T}{4\pi r^2} = \frac{E^2}{Z_o} \quad (Z_o=120\pi)$$

The directivity

$$D = \frac{P_{max}}{P_{av}} = 1$$

2.2 Electric field - Effective surface

The electric field is

$$E = \sqrt{P_r . Z_o} = \sqrt{\frac{W_T}{4\pi r^2} . Z_o} = \frac{\sqrt{30 . W_T}}{r} \quad (2.2)$$

Further the fictive antenna has no losses so:

$$\eta = \frac{G}{D} = 1 \quad with \;\; G=D=1$$

The effective surface becomes

$$A_{eff} = \frac{\lambda^2 G}{4\pi} = \frac{\lambda^2}{4\pi}$$

The importancy of this antenna is that all other antennas are definied (referred) with respect to the isotropic radiator.

3. Wire antennas

3.1 Hertz-dipole, short dipole, long dipole

3.1.1 Hertz-dipole

The radiated power and the field distributions from an antenna can be computed of a current distribution, assumed over the surface of the antenna. The simplest example is that of an ideal short linear element with current considered uniform over its length. More complex antennas can be considered to be composed with a large number of such small antennas.
The radiation pattern can be determined as a sum of individual fields.
The current element is in the *z* direction with its location the origin of a set of spherical coordinates (fig.3-1). Its length is L, with L very small ($L<\lambda/60$) compared with wavelength. By continuity, equal and opposite time-varying charges must exist on the two ends $\pm L/2$, so the elemnt is frequently called a *Hertzian dipole*.

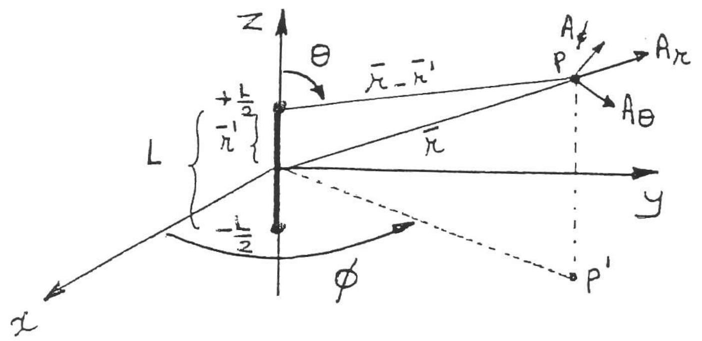


Fig.3-1: Hertzian dipole.

One way of finding fields once current is given requires only the rertaded potential (-vector) **A**. For any point Q at radius *r*, **A** becomes

$$A(r) = \int_V \frac{\mu}{4\pi} \frac{\overline{J(\bar{r}', t)} \cdot e^{-j\beta(\bar{r}-\bar{r}')}}{\|\bar{r}-\bar{r}'\|} \cdot dV \qquad (3.1)$$

$$\text{with } \mu\overline{H} = \overline{\nabla} \wedge \overline{A} \text{ and } \overline{\nabla} \wedge \overline{H} = j\omega\epsilon\overline{E} \qquad (3.2)$$

The current $\bar{I}=\bar{e}_z . I_m . e^{j\omega t}$ is supposed to be uniform over L. When I is constant we have

$$A(r) = A_z . \bar{e}_z \text{ with } A_z = \frac{\mu I_m}{4\pi r} . e^{-j\beta_r} . L \qquad (3.3)$$

Or, in the system of spherical coordinates

$$A_r = A_z.\cos\theta = \frac{\mu.I_m}{4\pi r}.e^{-j\beta r}.L\cos\theta \qquad (3.4)$$

$$A_\theta = -A_z.\sin\theta = -\frac{\mu.I_m}{4\pi r}.e^{-j\beta r}.L\sin\theta \qquad (3.5)$$

By application of

$$\overline{H} = \frac{1}{\mu}(\overline{\nabla} \wedge \overline{A})$$

we have

$$H_r=0,\, H_\theta=0,\, H_\phi=je^{-j\beta r}\frac{\beta I_m L\sin\theta}{4\pi r}(1+\frac{1}{j\beta r})$$

and with the Maxwell-equations

$$\overline{E} = \frac{1}{j\omega\epsilon}(\overline{\nabla} \wedge \overline{H})$$

we find

$$E_r=\frac{e^{-j\beta r}I_m L}{2\pi r^2}Z_o\cos\theta(1+\frac{1}{j\beta_r}) \qquad (3.6)$$

$$E_\theta=\frac{je^{-j\beta r}\beta}{4\pi r}I_m L Z_o\sin\theta(1+\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}) \qquad (3.7)$$

a) In the vincinity of the antenna (r<λ/6), the *near field* is valid:

$$H_\phi=\frac{e^{-j\beta r}I_m L\sin\theta}{4\pi r^2} \qquad (3.8)$$

$$E_r=\frac{e^{-j\beta r}I_m L Z_o\cos\theta}{j2\pi r^3\beta}$$

$$E_\theta=\frac{je^{-j\beta r}\beta I_m L Z_o\sin\theta}{4\pi r(j\beta r)^2}$$

b) Far from the antenna ($r>\lambda/6$), the *far field* is valid

$$H_\phi=\frac{je^{-j\beta r}\beta IL\sin\theta}{4\pi r} \qquad (3.9)$$

$$E_r=0 \qquad (2.10)$$

$$E_\theta=\frac{je^{-j\beta r}\beta ILZ_o\sin\theta}{4\pi r}$$

Because the **far field** is very important for telecommunication purposes, we'll consider this further in this course.

The power-density is

$$P_r=\frac{1}{2}(\bar{E}\wedge\bar{H})=\frac{I_m^2L^2\beta^2\sin^2\theta Z_o}{(4\pi r)^2}.\bar{e}_r \qquad (3.10)$$

$$=K.\sin^2\theta\bar{e}_r \qquad (2.11)$$

From this formula we can derive the (relative) radiation pattern for K=1

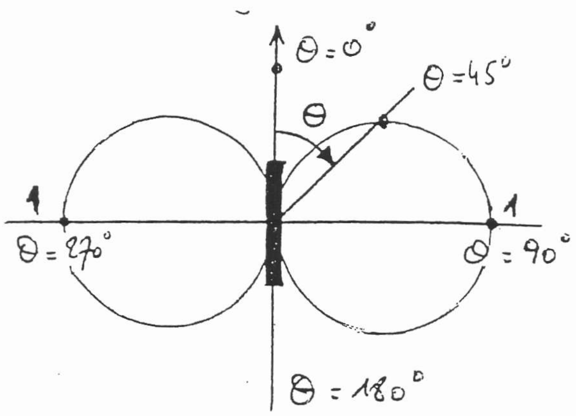


Fig.3-2: Relative radiation-pattern.

This is an omnidirectional pattern with beam-width $\theta_{3dB}=90°$.

The total power becomes

$$W_T=\int_\Omega P.dS=\frac{Z_o\beta^2I_m^2L^2}{\lambda^2} \quad with\ \beta=\frac{2\pi}{\lambda} \qquad (3.11)$$

The radiation-resistor is

$$R_r = \frac{W_T}{I_{av}^2} \approx 800\left(\frac{L}{\lambda}\right)^2$$

The directivity becomes

$$D = \frac{P_{max}}{P_{av}} = \frac{3}{2}$$

The effective height $h_{eff}=L$, while the effective surface is

$$A_{eff} = \frac{3}{8}\frac{\lambda^2}{\pi}$$

3.1.2 The short dipole

The length of this dipole $L \approx \lambda/30$ so the current is considered to be a linear function

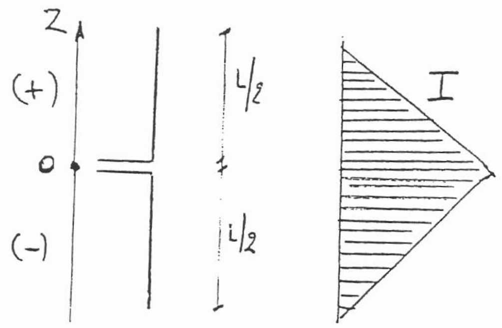


Fig.3-3: The short dipole - current distribution.

$$\text{for } z>0: \; I(z) = \frac{2}{L} I_m \left(\frac{L}{2} - z\right)$$
$$\text{for } z<0: \; I(z) = \frac{2}{L} I_m \left(\frac{L}{2} + z\right)$$

The current will be the half of the Hertz-dipole, then the retarded potential $\bar{A}$ and the field $(\bar{E},\bar{H})$ on a large distance become also the half,so

$$H_\phi = \frac{j e^{-j\beta r} \beta I_m L \sin\theta}{8\pi r} \qquad (3.12)$$

$$E_\theta = \frac{j e^{-j\beta r} \beta I_m L Z_o \sin\theta}{8\pi r}$$

And Poyntings theorem states:

$$P = \frac{1}{2}(\overline{E} \wedge \overline{H^*}) = \frac{I_m^2 L^2 \sin^2\theta \beta^2 Z_o}{128\pi^2 r^2}$$

The total radiated power becomes

$$W_T = \int_S P.dS = \int_0^\pi P.2\pi r^2 \sin\theta d\theta$$

thus

$$W_T = 10\pi^2 I_m^2 (\frac{L}{\lambda})^2$$

The radiation-resistor, determined from $W_T = I_{av}^2.R_r$, is

$$R_r = \frac{2W_T}{I_m^2} = 20\pi^2 \frac{L}{\lambda}^2$$

The radiation-diagram has the same pattern as the Hertz-dipole, so D and A_{eff} are the same

$$D = \frac{3}{2} \qquad A_{eff} = \frac{3\lambda^2}{8\pi}$$

The effective height $L_{eff} = L/2$

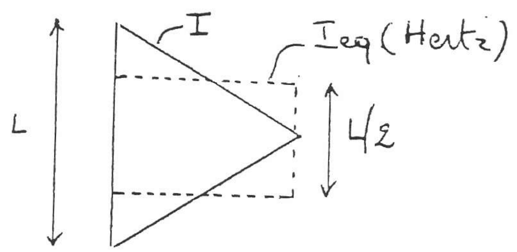


Fig.3-4: Effective height.

3.1.3 Long dipole

The current-distribution is now sinusoidal and can be considered as a superposition of a numbrous of elements of constant current. Consider the current-element I.dz then the electrical far field becomes

$$dE_\theta = \frac{I(z)\, e^{-j\beta r'} \sin\theta' j\beta Z_o}{4\pi r'} dz \qquad (3.13)$$

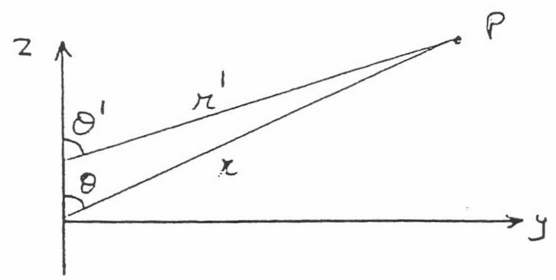


Fig.3-5: The long dipole.

with

$$for\ z>0 \quad I(z) = I_m \sin\left(\beta\left(\frac{L}{2} - z\right)\right)$$

$$for\ z<0 \quad I(z) = I_m \sin\left(\beta\left(\frac{L}{2} + z\right)\right)$$

Approximations for the far field

$$r' \approx r - z\cos\theta \quad ; \quad \theta \approx \theta' \quad ; \quad \left|\frac{1}{r'}\right| \approx \left|\frac{1}{r}\right|$$

The total electrical field can be written as

...

$$E_\theta = \frac{jZ_o I_m e^{-j\beta r}}{2\pi r} \cdot \underbrace{\frac{\cos\left(\beta\frac{L}{2}\cos\theta\right) - \cos\beta\frac{L}{2}}{\sin\theta}} \qquad (3.14)$$

with F(θ) the radiation-pattern.

The Poynting-vector results in

$$P_r = \frac{1}{2}\frac{|E_\theta|^2}{Z_o}$$

The total radiated power becomes

$$W_T=\int_S P.\,dS=\int_0^{\pi} P_r 2\pi r^2 \sin\theta\, d\theta$$

While the radiation resistor is

$$R_r=\frac{2W_T}{I_m^2}$$

Further

$$D=\frac{P_{max}}{W_T/4\pi r^2}\ ;\ A_{eff}=\frac{\lambda^2 D}{4\pi}\ ;\ h_{eff}=\frac{\lambda}{\pi}F(\theta)_{max}^{\frac{1}{2}}$$

Special cases:

- length L=odd(λ/2):

  Because

$$E_\theta=\frac{jZ_oI_m e^{-j\beta r}}{2\pi r}F(\theta)$$

$$F(\theta)=\frac{\cos(\beta\frac{L}{2}\cos\theta)-\cos\beta\frac{L}{2}}{\sin\theta}$$

and with

$$L=\frac{2n-1}{2}\lambda \quad n=1,2,\ldots$$

$$\Rightarrow F(\theta)=\frac{\cos(\frac{2n-1}{2}\pi\cos\theta)}{\sin\theta}$$

The last expression determines the radiation-pattern. We can find the zero's with

$$\frac{2n-1}{2}\pi\cos\theta=\pm(\frac{2k\pi-\pi}{2})\ \ or\ \cos\theta=\pm\frac{2k-1}{2n-1}$$

and the maximum of F(θ) with

$$\beta\frac{L}{2}\cos\theta=\pm k\pi\ \ or\ \cos\theta=\pm\frac{2k}{2n-1}$$

a) $\lambda/2$-dipole n=1

$F(\theta)=0$ if $\cos\theta=\pm 1$; $F(\theta)$ is max if $\cos\theta=0$;
We get

$$E_\theta=\frac{j60I_m e^{-j\beta r}}{r}\left(\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right)^2$$

and

$$P_r=\frac{1}{2}\frac{|E_\theta|^2}{Z_o}$$

$$W_T=36.6I_m^2$$

$$R_r=73,3\Omega$$

$D=1.64$; $A_{eff}=0.131\lambda^2$; $h_{eff}=0.319\lambda$;

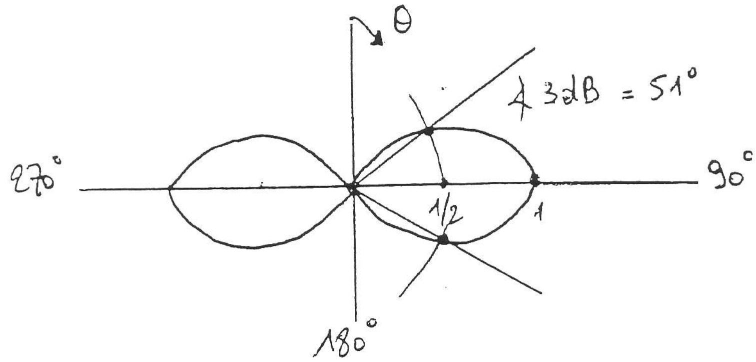


Fig.3-6: Radiation-pattern for n=1.

b) $3\lambda/2$ dipole n=2

zero's: $\cos\theta=\pm(2k-1)/3$
max. $F(\theta)$: $\cos\theta=\pm(2k)/3$

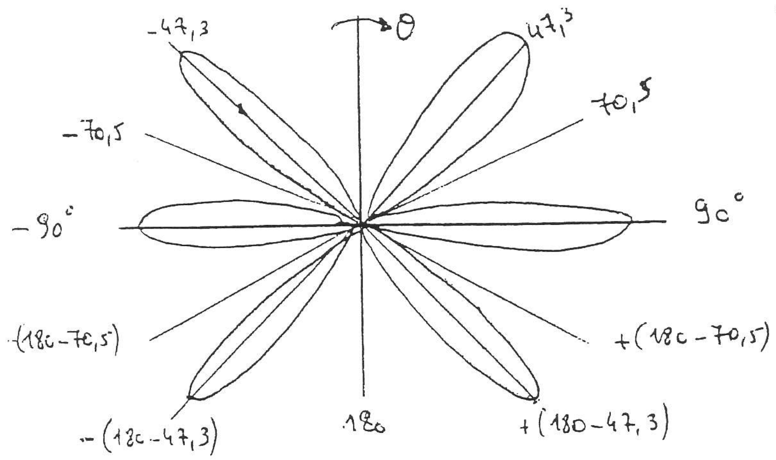


Fig.3-7: Radiation-pattern n=2.

3.2 Loop-antenna

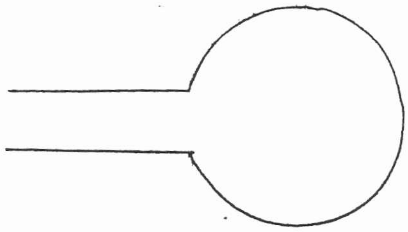


Fig.3-8: Loop-antenna.

To make the calculations easier we consider the loop built up as a rectangular structure, as shown in fig.3-2.

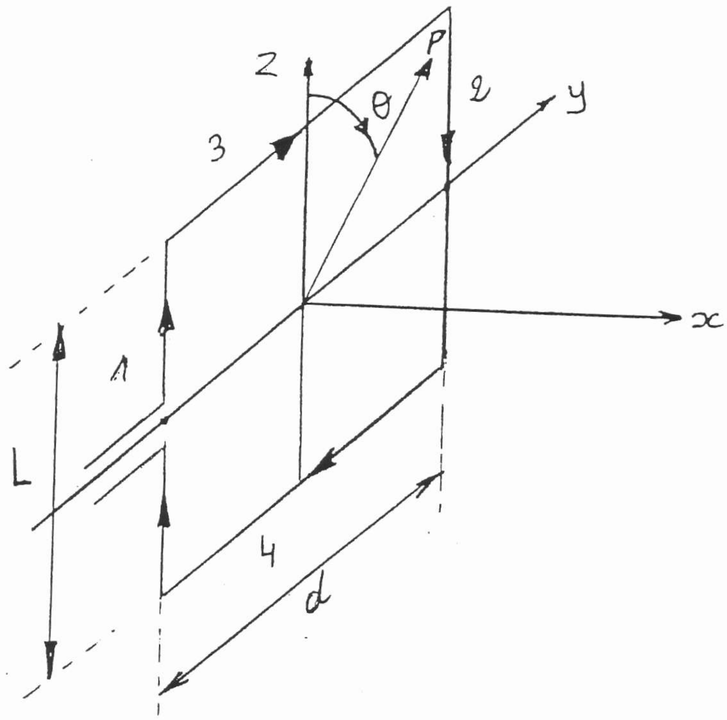


Fig.3-9: Rectangular loop-antenna.

In the x-y-section (θ=90°) the fields of 3 and 4 will compensate each other, while 1 and 2 give the resulting field. In the far field this becomes (for 1,2)

$$H_{\phi}=\frac{je^{-j\beta r}\beta I_m L\sin\theta}{4\pi r} \qquad (3.15)$$

$$E_{\theta}=\frac{je^{-j\beta r}\beta I_m L Z_o\sin\theta}{4\pi r}$$

We consider the x-y-section, conductors 1 and 2, together with the point P in the far field.

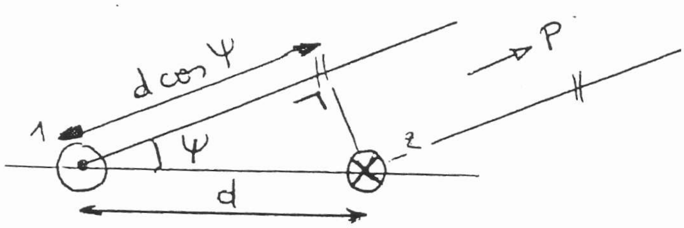


Fig.3-11: x-y-section of rectangular loop-antenna.

Resulting in a phase-shift ψ_1

$$\psi_1=\frac{2\pi.distance}{\lambda}=\frac{2\pi d\cos\psi}{\lambda}$$

Further the currents in 1 and 2 are opposite (ψ_2=180°), so we can write the resulting field as

$$E_R=E_o[1+e^{j(\psi_1+\psi_2)}] \quad or$$

....

$$|E_R|=\frac{\beta I_m L Z_o}{4\pi r}\sin\left(\frac{\pi d\cos\psi}{\lambda}\right)$$

and with $(\pi d)/\lambda$ very small this becomes

$$|E_R|=\frac{120 I_m L d\pi^2\cos\psi}{\lambda^2 r}=K.\cos\psi \qquad (3.16)$$

The power is

$$P=\frac{1}{2}\,\frac{|E_R|^2}{Z_o}=K^{2'}\cos^2\psi$$

The radiation-pattern for the x-y-section is shown in the figure on the next page.

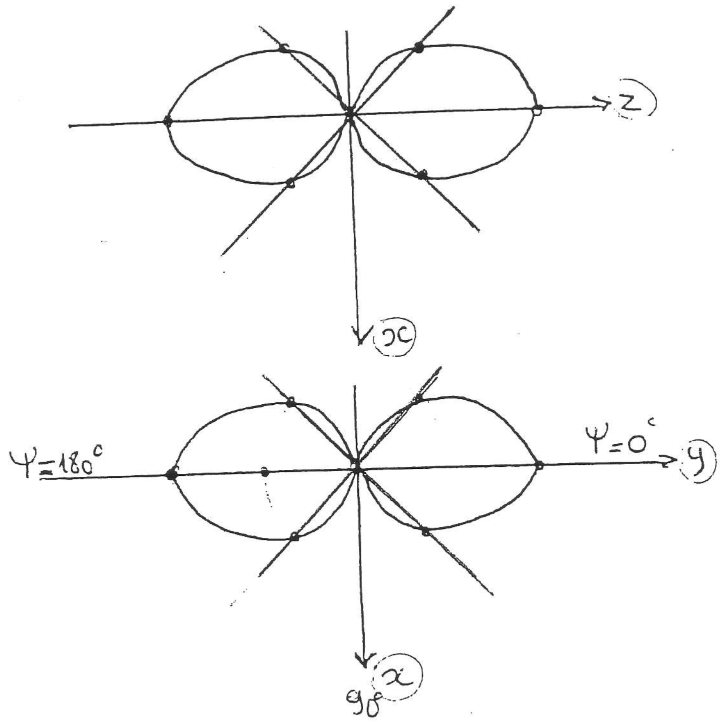


Fig.3-12: Radiation-pattern x-y-section loop-antenna.

When we make the same analyses above for the x-z-section and with conductors 3,4 we get the total radiation-pattern

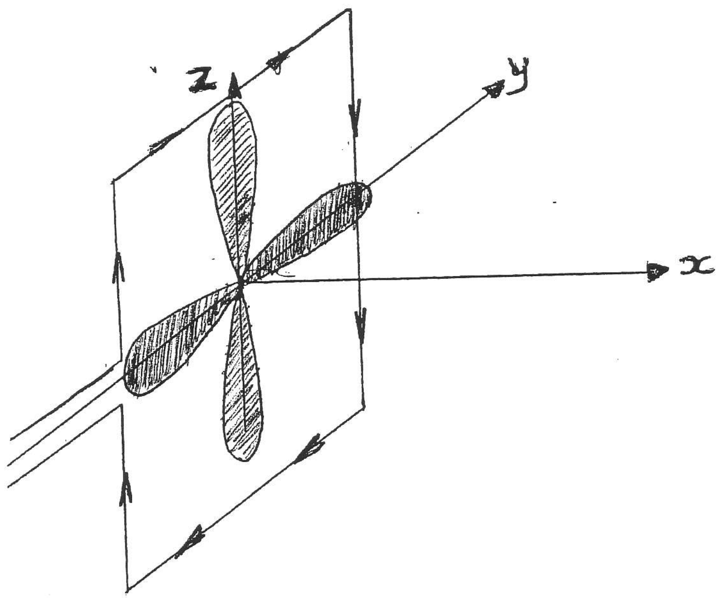


Fig.3-13: Radiation-pattern loop-antenna.

3.4 Static arrays

To increase the directivity, thus more radiation, power or energy in a specified direction, we place several antennas or elements near each other to become a constructive interference.
Consider a linear array

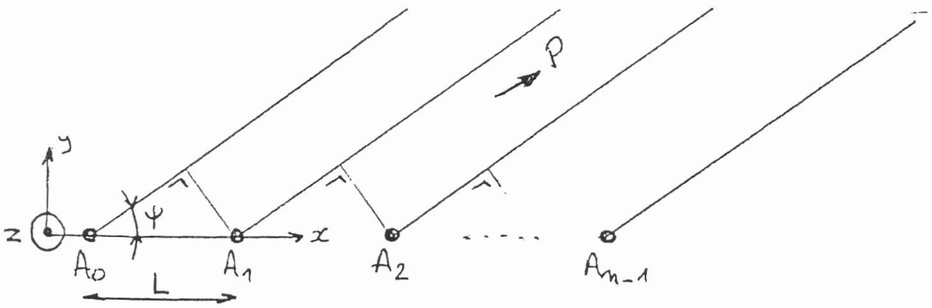


Fig.3-31: linear array of antennas.

with n radiators A, on a distance L from each other. The (far) field from one element is (see previous chapter)

$$dE_\theta = \frac{je^{-j\beta r}\beta IdL\sin\theta Z_o}{4\pi r} \qquad (3.23)$$

$$dH_\phi = \frac{je^{-j\beta r}\beta IdL\sin\theta}{4\pi r}$$

The wave, coming from A_1, must travel $L\cos\psi$ less than A_0.
A_1 has a positive phase-shift $\xi = 2\pi/\lambda . L\cos\psi$ with respect to A_0, A_2 2ξ to A_0, ... For n radiators we have

$$dE_{tot} = dE_\theta[1+e^{j\xi}+e^{2j\xi}+..+e^{(n-1)j\xi}]$$

When the n radiators have proper electrical phase shifts (via the feeders) θ_i, and if each radiator is fed with an individual magnitude with respect to the first radiator then

$$dE_{tot} = dE_\theta[1+A_1e^{j(\xi+\theta_1)}+..+A_{n-1}e^{j(n-1)\xi+j\theta_{n-1}}]$$

$$= dE_\theta . F \quad = dE_\theta|S|e^{j\psi} \qquad (3.24)$$

S=[F]='space-factor'. The radiation density becomes

$$P_r = S^2 . P_{ro} \quad with \quad S^2 = F.F^*$$

For 2 radiation-elements $S^2=1+2A\cos(\xi+\theta)+A^2$.

3.4.1 End-fire couplet

2 Identical radiators are placed on a distance $\lambda/4$. We feed the structure with a phase-shift of 90°. This means

$$A_0 = A_1;\ \xi = \frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\psi;\ \theta = -\frac{\pi}{2}$$

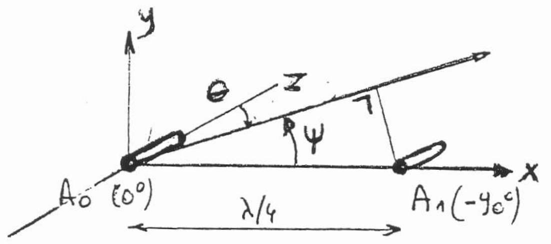


Fig.3-32: 2 Identical radiators on a distance $\lambda/4$.

A_1: the fields are in phase $(-\pi/2 + 2\pi/\lambda\ .\ \lambda/4)=0$
A_0: the fields are in opposite 180°
Mathematical this becomes

$$F = 1 + e^{-j\frac{\pi}{2}[1-\cos\psi]}$$

$$S^2 = F.F^* = 4\cos^2\left[\frac{1}{4}\pi(1-\cos\psi)\right] \qquad (3.24)$$

This results in a cardiod-graph in the xy-section

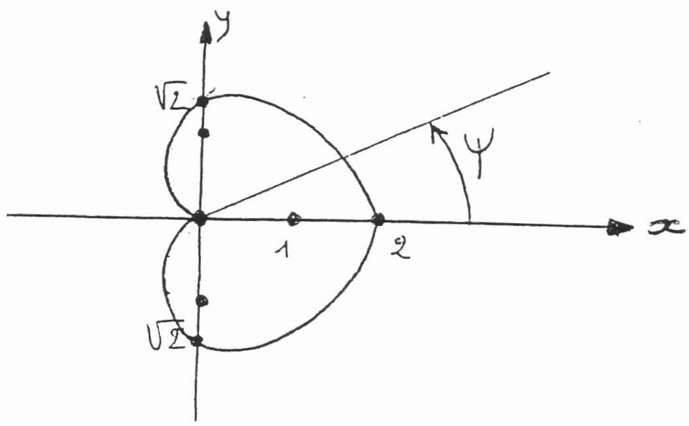


Fig.3-33: Cardiode-graph in the xy-section.

The antenna-gain G=6dB

3.4.2 Broadside couplet

Consider the same as above but distance between antennas equal to $\lambda/2$ and the elements are fed in phase.

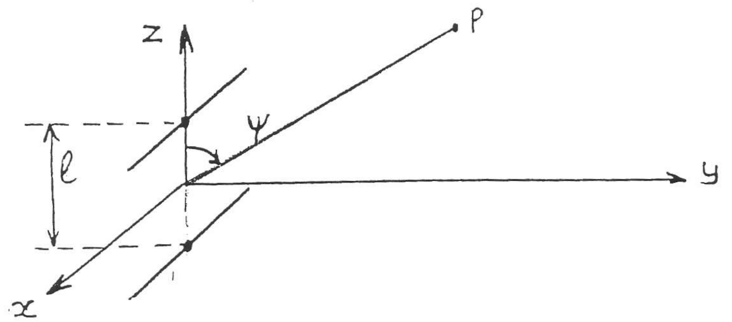


Fig.3-34: 2 Elements fed in phase on a distance $\lambda/2$.

Thus

$$A_0=A_1;\ \theta=0;\ \xi=\frac{2\pi}{\lambda}l\cos\psi;$$

$$F=1+e^{(j\frac{2\pi}{\lambda}l\cos\psi)} \qquad (3.25)$$

$$S+2\cos(\frac{\pi l}{\lambda}\cos\psi)$$

with

$$l=\frac{\lambda}{2},\quad S=2\cos(\frac{\pi}{2}\cos\psi)$$

$\psi=0$: $S=0$ → *radiation in z-direction=0*

$\psi=90^o$: $S=2$ → *radiation doubles w.r.t. one dipole in y-direction*

The power in y-direction increases with 4

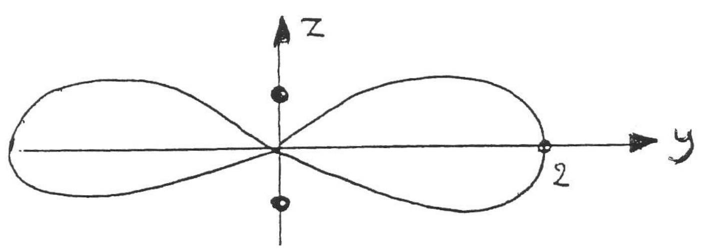


Fig.3-35: Broadside couplet.

3.4.3 Uniform broadside array

Consider the expression for F, using identical structures

$$F=1+e^{j\xi}+\ldots+e^{2j(n-1)\xi}$$

One can write this as

$$F=\frac{1-e^{jn\xi}}{1-e^{j\xi}}$$

$$S=\left|\frac{\sin\frac{n}{2}\xi}{\sin\frac{\xi}{2}}\right| \quad with\ \xi=\beta L\cos\psi \qquad (3.26)$$

When we extend the structure of 3.4.2 to an array, with length W and element-distance L, we obtain

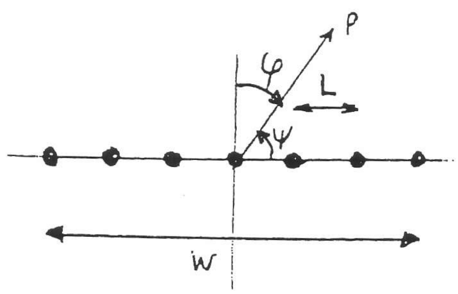


Fig. 3-36: array of elements.

Suppose $\theta_i=0$, then is the radiated power maximum for $\psi=90°$ (perpendicular to connection-line).

$$S=\left|\frac{\sin\frac{n}{2}\xi}{\sin\frac{\xi}{2}}\right|=\left|\frac{\sin(\frac{n\pi L}{\lambda}\cos\psi}{\sin(\frac{\pi L}{\lambda}\cos\psi)}\right| \qquad (3.27)$$

The pattern has main- and sidelobes, search the max and min and one can find some angles to plot the pattern on the next page.

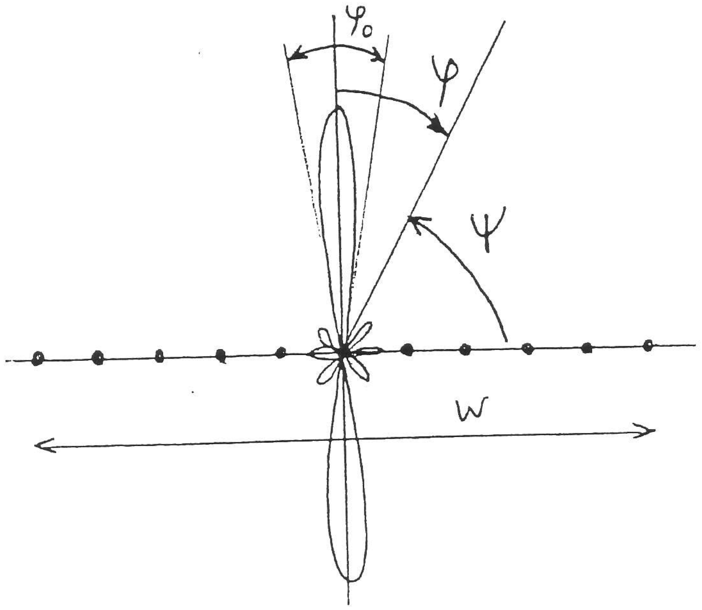


Fig3-37: Uniform broadside array.

3.4.4 Uniform end-fire array

This is an extension of the end-fire couplet

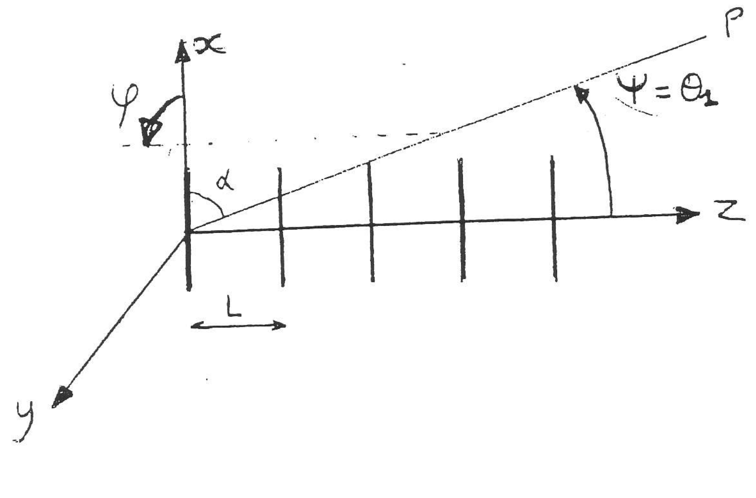


Fig.3-38: Uniform end-fire array.

$$S=\left|\frac{\sin\frac{n}{2}(\xi-\theta)}{\sin\frac{1}{2}(\xi-\theta)}\right|$$

$\theta=\beta L;\ \xi=\beta L\cos\psi;$

Resulting in

$$S=\left|\frac{\sin\frac{n\pi L}{\lambda}(1-\cos\theta)}{\sin\frac{\pi L}{\lambda}(1-\cos\theta)}\right| \qquad (3.28)$$

The radiation density

$P_r=S^2P_{ro}\sin^2\alpha$ ()

After some maths

$$P_r=15\pi\left(\frac{Idx}{\lambda}\right)^2\frac{\sin^2[\frac{n\pi L}{\lambda}(1-\cos\theta)]}{\sin^2[\frac{\pi L}{\lambda}(1-\cos\theta)]}(1-\sin^2\theta\cos^2\psi) \qquad (3.29)$$

For $L=\lambda/4$ and $\alpha=90°$ we have

$$P_r=K.\frac{\sin^2 n\pi/4(1-\cos\theta)}{\sin^2\pi/4(1-\cos\theta)}.1$$

This expression has a maximum for $\theta=0°$. We get the following pattern, for n=8 there are 7 lobes

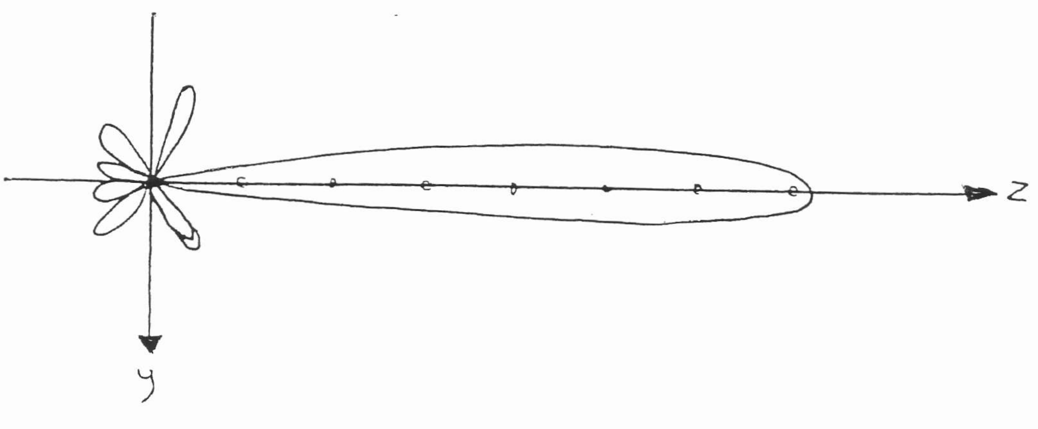


Fig.3-39: Uniform end-fire couplet, n=8, xz-section.